

SOME INHOMOGENEOUS MAGNETIZED VISCOUS FLUID COSMOLOGICAL MODELS WITH VARYING Λ

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Received (received date)

Revised (revised date)

Some cylindrically symmetric inhomogeneous viscous fluid cosmological models with electro-magnetic field are obtained. To get a solution a supplementary condition between metric potentials is used. The viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density. Without assuming any *ad hoc* law, we obtain a cosmological constant as a decreasing function of time. The behaviour of the electro-magnetic field tensor together with some physical aspects of the model are also discussed.

Keywords: Cosmology; Inhomogeneous models, Electro-magnetic fields

1. Introduction

Inhomogeneous cosmological models play an important role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such models have done by Tolman¹ and Bondi² who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub^{3,4} and later by Tomimura,⁵ Szekeres,⁶ Collins and Szafron,⁷ Szafron and Collins.⁸ Recently, Senovilla⁹ obtained a new class of exact solutions of Einstein's equation without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruis and Senovilla¹⁰ have separated out a fairly large class of singularity free models through a comprehensive study of general cylindrically symmetric metric with separable function of r and t as metric coefficients. Dadhich *et al.*¹¹ have established a link between the FRW model and the singularity free family by deducing the latter through a natural and simple inhomogenization and anisotropization of the former.

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Recently, Patel *et al.*¹² presented a general class of inhomogeneous cosmological models filled with non-thermalized perfect fluid by assuming that the background spacetime admits two space-like commuting killing vectors and has separable metric coefficients. Bali and Tyagi¹³ obtained a plane-symmetric inhomogeneous cosmological models of perfect fluid distribution with electro-magnetic field. Recently, Pradhan *et al.*¹⁴ have investigated a plane-symmetric inhomogeneous viscous fluid cosmological models with electro-magnetic field.

Models with a relic cosmological constant Λ have received considerable attention recently among researchers for various reasons (see Refs.^{15–19} and references therein). Some of the recent discussions on the cosmological constant “problem” and consequence on cosmology with a time-varying cosmological constant by Ratra and Peebles,²⁰ Dolgov^{21–23} and Sahni and Starobinsky²⁴ have pointed out that in the absence of any interaction with matter or radiation, the cosmological constant remains a “constant”. However, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researchers on this topic, are contained in Zeldovich,²⁵ Weinberg¹⁶ and Carroll, Press and Turner.²⁶ Recent observations by Perlmutter *et al.*²⁷ and Riess *et al.*²⁸ strongly favour a significant and positive value of Λ . Their finding arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ and these suggest Friedmann models with negative pressure matter such as a cosmological constant (Λ), domain walls or cosmic strings (Vilenkin,²⁹ Garavich *et al.*³⁰) Recently, Carmeli and Kuzmenko³¹ have shown that the cosmological relativistic theory (Behar and Carmeli³²) predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35} s^{-2}$. This value of “ Λ ” is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garavich *et al.*³⁰; Perlmutter *et al.*²⁷; Riess *et al.*²⁸; Schmidt *et al.*³¹) The main conclusion of these observations is that the expansion of the universe is accelerating.

Several ansatz have been proposed in which the Λ term decays with time (see Refs. Gasperini,^{34,35} Berman,³⁶ Freese *et al.*,¹⁹ Özer and Taha,¹⁹ Peebles and Ratra,³⁷ Chen and Hu,³⁸ Abdussattar and Viswakarma,³⁹ Gariel and Le Denmat,⁴⁰ Pradhan *et al.*⁴¹). Of the special interest is the ansatz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu,³⁸ which has been considered/modified by several authors (Abdel-Rahaman,⁴² Carvalho *et al.*,¹⁹ Waga,⁴³ Silveira and Waga,⁴⁴ Vishwakarma⁴⁵).

Most cosmological models assume that the matter in the universe can be described by ‘dust’ (a pressure-less distribution) or at best a perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe.^{46–48} It has been shown that bulk viscosity leads to inflationary like solution,⁴⁹ and acts like a negative energy field in an expanding universe.⁵⁰ Furthermore, there are several processes which

are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the Grand Unification Theories (GUT) phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn⁵¹ for a review on cosmological models with bulk viscosity). A number of authors have discussed cosmological solutions with bulk viscosity in various context.^{51–54}

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out Zeldovich *et al.*⁵⁵ Also Harrison⁵⁶ has suggested that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson-Walker model.⁵⁷ The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors.^{58–67} Strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Large-scale magnetic fields give rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and it decays slower than if the pressure was isotropic.^{68,69} Such fields can be generated at the end of an inflationary epoch.^{70–74} Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali⁷⁵ had obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Several authors^{76–81} have investigated Bianchi type I cosmological models with a magnetic field in different context.

Recently Singh *et al.*⁸² obtained some cylindrically symmetric inhomogeneous cosmological models for a perfect fluid distribution with electro-magnetic field. Motivated the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the general relativistic evolution magnetic inhomogeneities in presence of bulk viscous in an expanding universe. We do this by extending the work of Singh *et al.*⁸² by including an electrically neutral bulk viscous fluid as the source of matter in the energy-momentum tensor. This paper is organised as follows. The metric and the field equations are presented in section 2. In section 3 we deal with the solution of the field equations in presence of bulk viscous fluid. The sections 3.1, 3.2 and 3.3 contain the three cases as $K > 0$, $K < 0$ and $K = 0$ and also contain some physical aspects of these models respectively. Finally in section 4 concluding remarks have been given.

2. The metric and field equations

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2, \quad (1)$$

where the metric potential A is a function of t alone and B and C are functions of x and t . The energy momentum tensor in the presence of bulk stress has the form

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p}g_i^j + E_i^j, \quad (2)$$

where E_i^j is the electro-magnetic field given by Lichnerowicz⁸³ as

$$E_i^j = \bar{\mu} \left[|h|^2 \left(v_i v^j + \frac{1}{2} g_i^j \right) - h_i h^j \right] \quad (3)$$

and

$$\bar{p} = p - \xi v_{;i}^i \quad (4)$$

Here ρ , p , \bar{p} and ξ are the energy density, isotropic pressure, effective pressure, bulk viscous coefficient respectively and v^i is the flow vector satisfying the relation

$$g_{ij} v^i v^j = -1 \quad (5)$$

$\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{1}{\bar{\mu}} {}^*F_{ji} v^j \quad (6)$$

where ${}^*F_{ij}$ is the dual electro-magnetic field tensor defined by Synge⁸⁴ to be

$${}^*F_{ij} = \frac{\sqrt{-g}}{2} \epsilon_{ijkl} F^{kl} \quad (7)$$

F_{ij} is the electro-magnetic field tensor and ϵ_{ijkl} is the Levi-Civita tensor density. Here, the comoving coordinates are taken to be $v^1 = 0 = v^2 = v^3$ and $v^4 = \frac{1}{A}$. We consider the current to be flowing along the z -axis so that F_{12} is the only non-vanishing component of F_{ij} . The Maxwell's equations

$$F_{[ij,k]} = 0, \quad (8)$$

$$\left[\frac{1}{\bar{\mu}} F^{ij} \right]_{;j} = J^i, \quad (9)$$

require that F_{12} is a function of x alone. Here the semicolon represents a covariant differentiation. We assume that the magnetic permeability is a function of x and t . The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j, \quad (10)$$

for the line element (1) has been set up as

$$8\pi A^2 \left(\bar{p} + \frac{F_{12}^2}{2\bar{\mu} A^2 B^2} \right) = \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} - \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{B_1 C_1}{BC} - \Lambda A^2, \quad (11)$$

$$8\pi A^2 \left(\bar{p} + \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = \frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{C_{11}}{C} - \Lambda A^2, \quad (12)$$

$$8\pi A^2 \left(\bar{p} - \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = \frac{A_4^2}{A^2} - \frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{B_{11}}{B} - \Lambda A^2, \quad (13)$$

$$8\pi A^2 \left(\rho + \frac{F_{12}^2}{2\bar{\mu}A^2B^2} \right) = \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{B_{11}}{B} - \frac{C_{11}}{C} - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} - \Lambda A^2, \quad (14)$$

$$0 = \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A} \left(\frac{B_1}{B} + \frac{C_1}{C} \right), \quad (15)$$

where

$$\bar{p} = p - \frac{\xi}{A} \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right)$$

The suffixes 1 and 4 by the symbols A , B and C denote differentiation with respect to x and t respectively.

3. Solution of the field equations

From Equations (11), (12) and (13), we have

$$\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{B_{44}}{B} - \frac{B_4C_4}{BC} - \frac{C_{11}}{C} + \frac{B_1C_1}{BC} = 0 \quad (16)$$

$$\frac{8\pi F_{12}^2}{\bar{\mu}B^2} = -\frac{C_{44}}{C} + \frac{C_{11}}{C} + \frac{B_{44}}{B} - \frac{B_{11}}{B} \quad (17)$$

Equations (11) - (15) represent a system of five equations in eight unknowns A , B , C , ρ , p , F_{12} , Λ and ξ . To get a determinate solution, we need three extra conditions.

Firstly following Singh *et al.*⁸¹, we assume a supplementary condition between the metric potentials as

$$\frac{A_4}{A} = n \left(\frac{B_4}{B} - \frac{C_4}{C} \right), \quad (18)$$

and

$$\frac{C_{11}}{C} - \frac{B_1C_1}{BC} = K, \quad (19)$$

where n and K are arbitrary constants. Let us consider that

$$B = f(x)g(t) \quad (20)$$

$$C = h(x)k(t)$$

Using (18) and (20) in Equations (15), (16) and (17), we have

$$\frac{(n+1)\frac{\dot{k}}{k} - n\frac{\dot{g}}{g}}{(1-n)\frac{\dot{g}}{g} + n\frac{\dot{k}}{k}} = -\frac{f'/f}{h'/h} = \alpha \text{ (constant, say),} \quad (21)$$

$$(n-1)\frac{\ddot{g}}{g} - n\frac{\ddot{k}}{k} - \frac{\dot{g}\dot{k}}{gk} = K, \quad (22)$$

$$\frac{h''}{h} - \frac{f'h'}{fh} = K, \quad (23)$$

where prime and dot stand for differentiation with respect to x and t respectively. Integrating (21) leads to

$$\begin{aligned} f &= Lh^{-\alpha}, \\ g &= M_1 k^{\frac{n-n\alpha+1}{n-n\alpha+\alpha}}, \end{aligned} \quad (24)$$

where L and M_1 are constants of integration. Here we consider three cases according to the values of K .

3.1. Case (1): $K > 0$

Using (24) in Equations (22) and (23) and after making suitable transformation of coordinates, the geometry of the spacetime (1) reduces to the form

$$\begin{aligned} ds^2 &= a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)(dX^2 - dT^2) \\ &+ \cosh^{-\frac{2\alpha}{\alpha+1}}(K_2 X) \cosh^{\frac{N+\alpha-1}{N}}(K_1 T) dY^2 \\ &+ \cosh^{\frac{2}{\alpha+1}}(K_2 X) \cosh^{\frac{N-\alpha+1}{N}}(K_1 T) dZ^2, \end{aligned} \quad (25)$$

where

$$\begin{aligned} K_1 &= \sqrt{KN}, \\ K_2 &= \sqrt{K(\alpha+1)}, \\ N &= 2n\alpha - 2n - \alpha - 1, \\ a &= LM_1 a_2^{\frac{n(1-\alpha)}{n-n\alpha+\alpha}}. \end{aligned}$$

The effective pressure and density for the model (25) are given by

$$\begin{aligned} 8\pi\bar{p} &= 8\pi(p - \xi\theta) = \frac{1}{a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \times \\ &\left((n - n\alpha - \alpha + 2)K + \frac{\alpha^2 K}{\alpha + 1} \tanh^2(K_2 X) + K_3 \frac{K^2 \tanh^2(K_1 T)}{K_1^2} \right) - \Lambda, \end{aligned} \quad (26)$$

$$\begin{aligned} 8\pi\rho &= \frac{1}{a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \times \\ &\left((n\alpha - n - \alpha)K - \frac{\alpha(\alpha-2)K}{\alpha+1} \tanh^2(K_2 X) + K_3 \frac{K^2 \tanh^2(K_1 T)}{K_1^2} \right) + \Lambda, \end{aligned} \quad (27)$$

where

$$K_3 = n^2(3\alpha^2 - 4\alpha + 2) - 2n(\alpha - 1)(\alpha - 3) - 2(\alpha^2 - \alpha - 1).$$

Here θ is the scalar of expansion calculated for the flow vector v^i as

$$\theta = \frac{N_1 K \tanh(K_1 T)}{a K_1 \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T)}, \quad (28)$$

where

$$N_1 = 3n\alpha - 3n - \alpha - 1$$

For the specification of ξ , we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \quad (29)$$

where $\gamma(0 \leq \gamma \leq 1)$ is a constant.

Thus, given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density.^{85–87}

$$\xi(t) = \xi_0 \rho^m, \quad (30)$$

where ξ_0 and m are constants. If $m = 1$, Eq. (30) may correspond to a radiative fluid.⁸⁸ However, more realistic models⁸⁹ are based on m lying in the regime $0 \leq m \leq \frac{1}{2}$.

On using (30) in (26), we obtain

$$8\pi(p - \xi_0 \rho^m \theta) = \frac{1}{a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \times \left((n - n\alpha - \alpha + 2)K + \frac{\alpha^2 K}{\alpha + 1} \tanh^2(K_2 X) + K_3 \frac{K^2 \tanh^2(K_1 T)}{K_1^2} \right) - \Lambda, \quad (31)$$

3.1.1. *Model I:* $(\xi = \xi_0)$

When $m = 0$, Equation (30) reduces to $\xi = \xi_0$. With the use of Equations (27), (28) and (29), Equation (31) reduces to

$$8\pi(1 + \gamma)\rho = \frac{8\pi N_1 \xi_0 K \tanh(K_1 T)}{a K_1 \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T)} + \frac{2(1 - \alpha)(\alpha + 1)K K_1^2 + 2\alpha K K_1^2 \tanh^2(K_2 X) + 2(\alpha + 1)K^2 K_3 \tanh^2(K_1 T)}{(\alpha + 1)a^2 K_1^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)}. \quad (32)$$

Eliminating $\rho(t)$ between (27) and (32), we get

$$(1 + \gamma)\Lambda = \frac{8\pi N_1 \xi_0 K \tanh(K_1 T)}{a K_1 \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T)} +$$

$$\begin{aligned} & \frac{1}{(\alpha + 1)a^2 K_1^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \times \\ & \left[\{n - n\alpha - \alpha + 2 - (n\alpha - n - \alpha)\gamma\}(\alpha + 1)K K_1^2 + \right. \\ & \quad \left. \{\alpha + (\alpha - 2)\gamma\}\alpha K K_1^2 \tanh^2(K_2 X) \right. \\ & \quad \left. + (1 - \gamma)(\alpha + 1)K^2 K_3^2 \tanh^2(K_1 T) \right] \end{aligned} \quad (33)$$

3.1.2. *Model II:* $(\xi = \xi_0 \rho)$

When $m = 1$, Equation (30) reduces to $\xi = \xi_0 \rho$. With the use of (27), (28) and (29), Equation (31) reduces to

$$\begin{aligned} & 8\pi\rho \left[1 + \gamma - \frac{N_1 \xi_0 K \tanh(K_1 T)}{a K_1 \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T)} \right] = \\ & \frac{2(1 - \alpha^2)K K_1^2 + 2\alpha K K_1^2 \tanh^2(K_2 X) + 2(\alpha + 1)K^2 K_3 \tanh^2(K_1 T)}{(\alpha + 1)a^2 K_1^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)}. \end{aligned} \quad (34)$$

Eliminating $\rho(t)$ between (27) and (34), we get

$$\begin{aligned} & \Lambda \left[1 + \gamma - \frac{N_1 \xi_0 K \tanh(K_1 T)}{a K_1 \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T)} \right] = \\ & \frac{(n - n\alpha - \alpha + 2)(\alpha + 1)K K_1^2 + \alpha^2 K K_1^2 \tanh^2(K_2 X) + (\alpha + 1)K^2 K_3 \tanh^2(K_1 T)}{(\alpha + 1)a^2 K_1^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} - \\ & \frac{(n\alpha - n - \alpha)(\alpha + 1)K K_1^2 - \alpha(\alpha - 2)K K_1^2 \tanh^2(K_2 X) + (\alpha + 1)K^2 K_3 \tanh^2(K_1 T)}{(\alpha + 1)a^2 K_1^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \times \\ & \left[\gamma - \frac{N_1 \xi_0 K \tanh(K_1 T)}{a K_1 \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T)} \right] \end{aligned} \quad (35)$$

From Equations (33) and (35), we observe that the cosmological constant is a decreasing function of time and it approaches a small value as time progresses (i.e., the present epoch), which explains the small value of Λ at present. Figure 1 clearly shows this behaviour of Λ as decreasing function of time in both the models I and II.

Some Physical Aspects of the Models

We shall now give the expressions for kinematical quantities and the components of conformal curvature tensor. With regard to the kinematical properties of the velocity vector v^i in the metric (25), a straight forward calculation leads to the following expressions for the shear of the fluid.

$$\sigma_{11} = \frac{a(\alpha + 1)K}{3K_1} \cosh^{\frac{n(\alpha-1)}{N}}(K_1 T) \tanh(K_1 T), \quad (36)$$

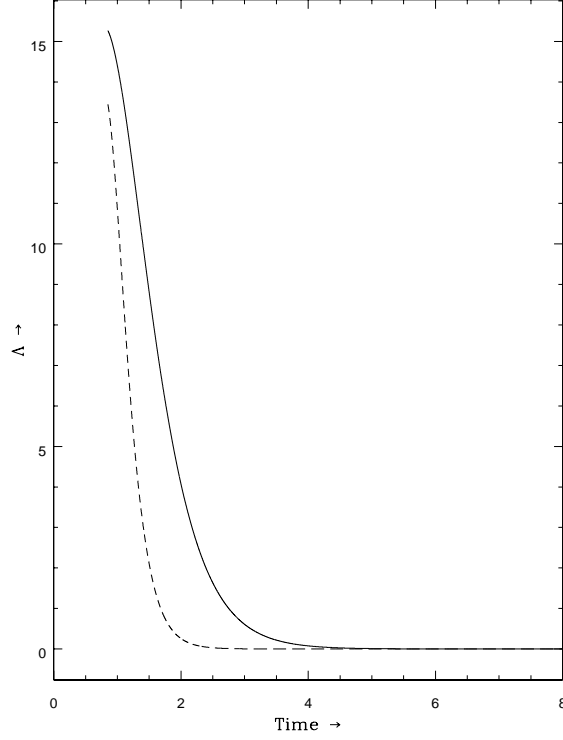


Figure 1: The plot of cosmological constant (Λ) with time for model $K > 0$ with parameters $\alpha = 1.5$, $\gamma = 0.5$, $K = 1$, $a = 1$, $\xi_0 = 1$, solid line for power index $m = 0$, and dashed line for $m = 1$ respectively.

$$\sigma_{22} = \frac{(\alpha - 2)K}{3aK_1} \cosh^{\frac{-2\alpha}{\alpha+1}}(K_2X) \cosh^{\frac{n\alpha-n-2}{N}}(K_1T) \tanh(K_1T), \quad (37)$$

$$\sigma_{33} = \frac{(1 - 2\alpha)\sqrt{K}}{3aK_1} \cosh^{\frac{2}{\alpha+1}}(K_2X) \cosh^{\frac{n\alpha-n-2\alpha}{N}}(K_1T) \tanh(K_1T), \quad (38)$$

$$\sigma_{44} = \frac{aN_1K}{3K_1} \cosh^{\frac{n(\alpha-1)}{N}}(K_1T) \tanh(K_1T), \quad (39)$$

where all other components, the rotation tensor ω and acceleration vanish. The expansion scalar θ has already been given by (49). Since $\dot{v}_i = v_{i;j}v^j = 0$, the motion is geodetic. The non-vanishing physical components of conformal curvature tensor are given by

$$C_{12}^{12} = C_{34}^{34} = \frac{1}{6a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1T)} [4\alpha K \tanh^2(K_2X) - (\alpha + 1)K +$$

$$\left\{ n^2(\alpha - 1)^2 - n(5\alpha^2 - 4\alpha + 1) + 2\alpha \right\} \frac{K^2 \tanh^2(K_1 T)}{K_1^2} \Big] \quad (40)$$

$$C_{13}^{13} = C_{24}^{24} = \frac{1}{6a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \left[4K - 2\alpha K \tanh^2(K_2 X) + \right. \\ \left. \{ 2n(\alpha - 2)(\alpha - 1) + 2\alpha \} \frac{K^2 \tanh^2(K_1 T)}{K_1^2} \right] \quad (41)$$

$$C_{14}^{14} = C_{23}^{23} = \frac{1}{6a^2 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \left[\frac{2(n\alpha^2 - 2\alpha - 1)K^2}{K_1^2} \tanh^2(K_1 T) - \right. \\ \left. 2\alpha K \tanh^2(K_2 X) - 2K \right] \quad (42)$$

$$C_{12}^{24} = C_{13}^{34} = \frac{n(1 - \alpha)\sqrt{(\alpha + 1)}K^{\frac{3}{2}} \tanh(K_1 T) \tanh(K_2 X)}{2a^2 K_1 \cosh^{\frac{2n(\alpha-1)}{N}}(K_1 T)} \quad (43)$$

The non-vanishing component F_{12} of electro-magnetic field tensor and J^2 , the component of charge current density, are obtained as

$$F_{12}^2 = \left(\frac{\bar{\mu}\alpha K}{4\pi} \right) \operatorname{sech}^{\frac{2(2\alpha+1)}{\alpha+1}}(K_2 X) \cosh^{\frac{N+\alpha-1}{N}}(K_1 T) \quad (44)$$

$$J^2 = \frac{1}{4a^2} \sqrt{\frac{\alpha K}{\bar{\mu}\pi(\alpha + 1)}} \cosh^{-\frac{1}{\alpha+1}}(K_2 X) \operatorname{sech}^{\frac{N_1+\alpha}{N}}(K_1 T) \\ \left[\bar{\mu}' \sqrt{(\alpha + 1)} - 2\alpha \sqrt{K} \bar{\mu} \tanh(K_2 X) \right] \quad (45)$$

The source of electro-magnetic field exist, when matter and charge are present and does not exist when matter is absent. The models represent expanding, shearing, non-rotating and Petrov type I non-degenerate in general, in which the flow is geodesic. The model starts expanding at $T > 0$ and goes on expanding indefinitely. It is observed that the expansion is minimum at $T = 0$. Since $\lim_{T \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$, the models do not approach isotropy for large values of T . The pressure contrast $\frac{p_x}{p}$ and density contrast $\frac{\rho_x}{\rho}$ tends to zero for large values of T which shows that inhomogeneity dies out for large values of T . When $\alpha = -1$ the model reduces to its homogeneous form. It is remarkable to mention here that at the time of minimum expansion matter density dominates over the expansion in the model whereas at the time of maximum expansion, expansion in the model dominates over matter density.

3.2. Case (2) : $K < 0$, let $K = -M$, $M > 0$

Using (24) in Equations (22) and (23) and after making suitable transformation of co-ordinates, the geometry of the spacetime (1) reduces to the form

$$ds^2 = b^2 \cos^{\frac{2n(\alpha-1)}{N}}(K_4 T) (dX^2 - dT^2) \\ + \cos^{-\frac{2\alpha}{\alpha+1}}(K_5 X) \cos^{-\frac{(N+\alpha-1)}{N}}(K_4 T) dY^2$$

$$+ \cos^{\frac{2}{\alpha+1}}(K_5 X) \cos^{\frac{-(N+\alpha-1)}{N}}(K_4 T) dZ^2, \quad (46)$$

where

$$\begin{aligned} K_4 &= \sqrt{MN}, \\ K_5 &= \sqrt{M(\alpha+1)}, \\ b &= LM_1 a'_2 \frac{n(1-\alpha)}{n-n\alpha+\alpha} \end{aligned}$$

The effective pressure and density for the model (46) are given by

$$\begin{aligned} 8\pi\bar{p} &= 8\pi(p - \xi\theta) = \frac{1}{b^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)} \times \\ &\left[-(N+1)M + \frac{2\alpha^2 M}{\alpha+1} \tan^2(K_5 X) + K_6 \frac{M^2 \tan^2(K_4 T)}{K_4^2} \right] - \Lambda, \end{aligned} \quad (47)$$

$$\begin{aligned} 8\pi\rho &= \frac{1}{b^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)} \times \\ &\left[(2-\alpha)M - \frac{\alpha(\alpha-2)M}{\alpha+1} \tan^2(K_5 X) + K_7 \frac{M^2 \tan^2(K_4 T)}{K_4^2} \right] + \Lambda, \end{aligned} \quad (48)$$

where

$$K_6 = n(\alpha-1)(5n-5n\alpha+4\alpha+4) - \alpha^2 - \alpha - 1,$$

$$K_7 = n(\alpha-1)(3n\alpha-3n-4\alpha) + \alpha^2 + \alpha - 1.$$

and θ is the scalar of expansion calculated for the flow vector v^i as

$$\theta = \frac{N_1 M \tan(K_4 T)}{b K_4 \cos^{\frac{n(1-\alpha)}{N}}(K_4 T)} \quad (49)$$

3.2.1. *Model I:* $(\xi = \xi_0)$

When $m = 0$, Equation (30) reduces to $\xi = \xi_0$. In this case Equation (47) with the use of (29), (48) and (49) reduces to

$$\begin{aligned} 8\pi(1+\gamma)\rho &= \frac{8\pi N_1 \xi_0 M \tan(K_4 T)}{b K_4 \cos^{\frac{n(1-\alpha)}{N}}(K_4 T)} + \\ &\frac{-(N+1)(\alpha+1)M K_4^2 + \alpha(\alpha+2)M K_4^2 \tan^2(K_5 X) + (K_6 + K_7)(\alpha+1)M^2 \tan^2(K_4 T)}{(\alpha+1)b^2 K_4^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)}. \end{aligned} \quad (50)$$

Eliminating $\rho(t)$ between (48) and (50), we get

$$(1+\gamma)\Lambda = \frac{8\pi N_1 \xi_0 M \tan(K_4 T)}{b K_4 \cosh^{\frac{n(1-\alpha)}{N}}(K_4 T)} +$$

$$\begin{aligned} & \frac{1}{(\alpha + 1)b^2 K_4^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)} \times \\ & \left[\{2(n - \gamma) - \alpha(2n - \gamma) + \alpha\}(\alpha + 1)MK_4^2 + \right. \\ & \quad \left. \{\alpha^2(2 + \gamma) - 2\alpha\gamma\}MK_4^2 \tan^2(K_5 X) \right. \\ & \quad \left. + (K_6 - \gamma K_7)(\alpha + 1)M^2 \tan^2(K_4 T) \right] \end{aligned} \quad (51)$$

3.2.2. *Model II:* $(\xi = \xi_0 \rho)$

When $m = 1$, Equation (30) reduces to $\xi = \xi_0 \rho$. In this case Equation (47) with the use of (29), (48) and (49), reduces to

$$\begin{aligned} & 8\pi\rho \left[1 + \gamma - \frac{N_1 \xi_0 M \tan(K_4 T)}{bK_4 \cos^{\frac{n(1-\alpha)}{N}}(K_4 T)} \right] = \\ & \frac{-2(n+1)(\alpha^2 - 1)MK_4^2 + \alpha(\alpha + 2)MK_4^2 \tan^2(K_5 X) + (K_6 + K_7)(\alpha + 1)M^2 \tan^2(K_4 T)}{(\alpha + 1)b^2 K_4^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)}. \end{aligned} \quad (52)$$

Eliminating $\rho(t)$ between (48) and (52), we get

$$\begin{aligned} & \Lambda \left[1 + \gamma - \frac{N_1 \xi_0 M \tan(K_4 T)}{bK_4 \cos^{\frac{n(1-\alpha)}{N}}(K_4 T)} \right] = \\ & \frac{(2n - 2n\alpha - \alpha)(\alpha + 1)MK_4^2 + 2\alpha^2 MK_4^2 \tan^2(K_5 X) + (\alpha + 1)K_6 M^2 \tan^2(K_4 T)}{(\alpha + 1)b^2 K_4^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)} - \\ & \frac{(2 - \alpha)(\alpha + 1)MK_4^2 - \alpha(\alpha - 2)MK_4^2 \tan^2(K_5 X) + (\alpha + 1)M^2 K_7 \tan^2(K_4 T)}{(\alpha + 1)b^2 K_4^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4 T)} \times \\ & \left[\gamma - \frac{N_1 \xi_0 M \tan(K_4 T)}{bK_4 \cos^{\frac{n(1-\alpha)}{N}}(K_4 T)} \right] \end{aligned} \quad (53)$$

Our study for the case $K < 0$ shows constant value of cosmological constant (Λ) for large values of time and do not decrease with time (this means that the universe is not expanding or may be steady state condition). In this case, detailed study shows that the scalar of expansion θ does not increase with time. Our study is inconsistent with work done by Singh *et al.*⁸² The Singh *et al.* claim that the universe is expanding which does not match with our result. Also the claim of minimum and maximum expansion rate in θ is reflection of periodicity of trigonometric functions involved there. We are trying to find feasible interpretation and situations relevant to $K < 0$. Further study is in progress.

Some Physical Aspects of the Models

The non-vanishing components of shear tensor σ_{ij} are obtained as

$$\sigma_{11} = \frac{b(\alpha + 1)M}{3K_4} \sec^{\frac{n(\alpha-1)}{N}}(K_4 T) \tan(K_4 T), \quad (54)$$

$$\sigma_{22} = \frac{(\alpha - 2)M}{3bK_4} \sec^{\frac{2\alpha}{\alpha+1}}(K_5X) \sec^{\frac{n\alpha-n-2}{N}}(K_4T) \tan(K_4T), \quad (55)$$

$$\sigma_{33} = \frac{(1 - 2\alpha)M}{3bK_4} \sec^{-\frac{2}{\alpha+1}}(K_5X) \sec^{\frac{n\alpha-n-2\alpha}{N}}(K_4T) \tan(K_4T), \quad (56)$$

$$\sigma_{44} = \frac{N_1M}{3bK_4} \sec^{\frac{n(\alpha-1)}{N}}(K_4T) \tan(K_4T) \quad (57)$$

The expansion scalar θ has already been given by (49). The rotation ω is identically zero. Since $v_i = v_{i;j}V^j = 0$, the motion is geodesic.

The non-vanishing physical components of conformal curvature tensor are given by

$$C_{12}^{12} = C_{34}^{34} = \frac{1}{6b^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4T)} \left[(4\alpha + n - n\alpha)M + 4\alpha M \tan^2(K_5X) + \{n(\alpha - 1)(3n - 3n\alpha + 5\alpha - 1) - 4\alpha^2 + 2\} \frac{M \tan^2(K_4T)}{N} \right] \quad (58)$$

$$C_{13}^{13} = C_{24}^{24} = \frac{1}{6b^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4T)} \left[\frac{\{2n(1 - \alpha)(\alpha - 2) + \alpha^2 - 2\}}{K_4^2} M^2 \tan^2(K_4T) - 2\alpha M \sec^2(K_5X) \right] \quad (59)$$

$$C_{14}^{14} = C_{23}^{23} = \frac{1}{6b^2 \cos^{\frac{2n(1-\alpha)}{N}}(K_4T)} \left[\frac{2(\alpha^2 - n\alpha^2 + n + 1)}{K_4^2} M^2 \tan^2(K_4T) - 2\alpha M \sec^2(K_5X) \right] \quad (60)$$

$$C_{12}^{24} = C_{13}^{34} = \frac{n(\alpha - 1)\sqrt{(\alpha + 1)}M^{\frac{3}{2}} \tan(K_4T) \tan(K_5X)}{2b^2 K_4 \cos^{\frac{2n(1-\alpha)}{N}}(K_4T)} \quad (61)$$

The non-vanishing component F_{12} of electro-magnetic field tensor and J^2 , the component of charge current density, are obtained as

$$F_{12}^2 = \left(\frac{\bar{\mu}M}{4\pi} \right) \sec^{\frac{2\alpha}{\alpha+1}}(K_5X) \sec^{\frac{N+\alpha-1}{N}}(K_4T) \left[\frac{\{\alpha^2(2n - 1) - 2n(2\alpha - 1) + 1\}}{K_4^2} M \tan^2(K_4T) - \alpha \tan^2(K_5X) - 1 \right] \quad (62)$$

$$J^2 = \frac{1}{4b^2} \sqrt{\frac{M}{\bar{\mu}\pi H_1(\alpha + 1)}} \operatorname{sech}^{\frac{-(N_1+\alpha)}{N}}(K_4T) \sec^{\frac{-\alpha}{\alpha+1}}(K_5X) \left[2\bar{\mu}\alpha(\alpha + 1)\sqrt{M} \tan(K_5X) \sec^2(K_5X) - \bar{\mu}'H_1\sqrt{(\alpha + 1)} + 2\bar{\mu}H_1\sqrt{M} \tan(K_5X) \right] \quad (63)$$

where

$$H_1 = \left[\frac{\{\alpha^2(2n - 1) - 2n(2\alpha - 1) + 1\}}{K_4^2} M^2 \tan^2(K_4T) - \alpha \tan^2(K_5X) - 1 \right]$$

The models represent shearing, non-rotating and Petrov type I non-degenerate in general, in which the flow is geodetic. The model starts expanding at $T > 0$ but the initial expansion is slow. When T is closer to $\frac{\pi}{2K_4}$, it has stiff rise in the expansion then decreases. This shows the case of oscillation. It is observed that the expansion is minimum at $T = 0$ or $T = \frac{\pi}{K_4}$. The large values of θ near $T = \frac{\pi}{2K_4}$ is reflection of trigonometric property. But expansion remains finite. Since $\lim_{T \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$, hence the models do not approach isotropy for large values of T . In this case inhomogeneity also dies out for large value of T .

3.3. Case (3): $K = 0$

Using (24) in Equations (22) and (23) and after making suitable transformation of co-ordinates, the geometry of the spacetime (1) reduces to the form

$$ds^2 = c^2 T^{\frac{2n(\alpha-1)}{N}} (dX^2 - dT^2) + X^{\frac{-2\alpha}{\alpha+1}} T^{\frac{N+\alpha-1}{N}} dY^2 + X^{\frac{-2}{\alpha+1}} T^{\frac{N-\alpha+1}{N}} dZ^2, \quad (64)$$

where

$$c = LM_1 a_2''^{\frac{n(1-\alpha)}{n-n\alpha+\alpha}}.$$

It is observed that the model (64) starts to expand from its singularity stage i.e. at $T = 0$ and goes expanding indefinitely when $T \rightarrow \infty$. The model represents expanding, shearing, non-rotating and Petrov type I non-degenerate in general in which the flow is geodetic. This model also does not approach isotropy for large values of T . It is also observed here at the time of initial singularity the matter density dominates over the expansion of the model.

4. Conclusions

We have obtained a new class of cylindrically symmetric inhomogeneous cosmological models electro-magnetic bulk viscous fluid as the source of matter. Generally the models represent expanding, shearing, non-rotating and Petrov type-I non-degenerate in which the flow vector is geodetic. In all these models, we observe that they do not approach isotropy for large values of time. It is concluded that (i) if $\alpha > 0$ then for the cases $K > 0$ or the values of K tending to 0, the models have point type singularity at the time of maximum expansion and (ii) if $0 < \alpha < 1$ then they have infinite singularity. Whereas the case $K < 0$ is just opposite of the case $K \geq 0$. In all the cases the spacetime is conformally flat for large values of T and at the time of minimum expansion matter density dominates over expansion. For $K = 0$ the material energy is more dominant over magnetic energy. The cosmological constant in all models given in section 3 are decreasing function of time and they all approach a small value as time increases (i.e., the present epoch) except the case $K < 0$. The values of cosmological “constant” for these models are found to be small and positive which are supported by the results from recent supernovae observations recently obtained by the High - z Supernova Team and Supernova Cosmological Project (Garnavich *et al.*³⁰; Perlmutter *et al.*²⁷; Riess *et al.*²⁸; Schmidt *et al.*³³). Thus, with our

approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where *ad hoc* laws were used to arrive at a mathematical expressions for the decaying vacuum energy. Thus our models are more general than those studied earlier. We would like to find feasible situations for $K < 0$. Our strong point of this model is that it in-cooperates matter density naturally and so makes feasible model which can be in-cooperates the physical constraints.

Acknowledgements

A. Pradhan and Kanti R. Jotania thank to the Inter-University Centre for Astronomy and Astrophysics, Pune, India for providing facility under Associateship Programme where part of this work was carried out.

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